6-2 day 2 Integration by u-Substitution

Learning Objectives:

I can evaluate an integral using u-substitution.

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Ex1. Evaluate

1.) $\int (2x+1)^5 dx$

$\underbrace{u = 2x + 1}$

$\frac{du}{dx} = 2$

$du = 2\,dx$

$dx = \frac{du}{2}$

$\int u^5 \frac{du}{2}$

$\frac{1}{2}\int u^5 du = \frac{1}{2} \cdot \frac{1}{6} u^6 + C$

$\frac{1}{12} u^6 + C$

$\boxed{\frac{1}{12}(2x+1)^6 + C}$

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2.) $\int \sqrt{3x+8}\,dx$ $\quad u = 3x+8$

$\frac{du}{dx} = 3 \quad dx = \frac{du}{3} \quad \frac{1}{3}\int \sqrt{u}\,du$

$\frac{1}{3}\cdot\frac{2}{3}u^{3/2}+c \quad \frac{2}{9}u^{3/2}+c$

$\frac{2}{9}(3x+8)^{3/2}+C$

3.) $\int \sin\left(3x+\frac{\pi}{2}\right)dx$

$u=3x+\frac{\pi}{2}$ $\frac{du}{dx}=3$ $dx=\frac{du}{3}$

$\int \sin u\, dx$ $\frac{1}{3}\int \sin u\, du$

$\frac{1}{3}\cdot -\cos u$ $-\frac{1}{3}\cos\left(3x+\frac{\pi}{2}\right)+C$

4.) $\int x\sqrt[3]{2x^2+5}\,dx$

$\int x \cdot \sqrt[3]{2x^2+5}\,dx$

$\int x(2x^2+5)^{\frac{1}{3}}dx$ $u=2x^2+5$

$\frac{du}{dx}=4x$ $dx=\frac{du}{4x}$ $\int \cancel{x}\cdot u^{\frac{1}{3}}\frac{du}{4\cancel{x}}$

$\frac{1}{4}\int u^{\frac{1}{3}}\,du$ $\frac{1}{4}\cdot\frac{3}{4}u^{\frac{4}{3}}+C$

$$\boxed{\frac{3}{16}(2x^2+5)^{\frac{4}{3}}+C}$$

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5.) $\int (3x^2+6x+2)^2(x+1)dx$ $u=3x^2+6x+2$

$\frac{du}{dx}=6x+6$ $dx=\frac{du}{6x+6}$

$\int u^2(x+1)dx$ $\int u^2\cancel{(x+1)}\frac{du}{6\cancel{(x+1)}}$

$\frac{1}{6}\int u^2 du$ $\frac{1}{6}\cdot\frac{1}{3}u^3+C$ $\frac{1}{18}(3x^2+6x+2)^3+C$

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6.) $\int \frac{3x-4}{3x^2-8x+3}\,dx$ $\int \frac{1}{3x^2-8x+3} \cdot (3x-4)\,dx$

$\int (3x^2-8x+3)^{-1} \cdot (3x-4)\,dx$

$u = 3x^2-8x+3$

$\int u^{-1} \cdot \cancel{(3x-4)} \frac{du}{2\cancel{(3x-4)}}$

$\frac{du}{dx} = 6x-8$

$dx = \frac{du}{2(3x-4)}$

$= \frac{1}{2} \int u^{-1}\,du$

$\frac{1}{2} \ln|u| + C$

$\frac{1}{2} \ln|3x^2-8x+3| + C$

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8.) $\int \cos^2(2x)\sin(2x)dx$

$u = \cos(2x)$

$\frac{du}{dx} = -2\sin(2x)$

$dx = \frac{du}{-2\sin(2x)}$

$\int u^2 \cdot \cancel{\sin(2x)} \frac{du}{-2\cancel{\sin(2x)}}$

$-\frac{1}{2}\int u^2 \, du$

$-\frac{1}{2} \cdot \frac{1}{3}u^3 + C$

$-1/6\, u^3 + C$

$-1/6 \cos^3(2x) + C$

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9.) $\int_{e}^{e^3} \frac{\ln x}{x} dx$

$\int_{e}^{e^3} (\ln x)^1 \cdot \frac{1}{x}\, dx$

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$x\,du = dx$

$\int u^1 \cdot \frac{1}{\cancel{x}} \cdot \cancel{x}\, du$

$\int u\, du$

$= \frac{1}{2}u^2$

must change back to "x" b/c the limits are $x = e$ to $x = e^3$

$= \frac{1}{2}(\ln x)^2 \Big|_{e}^{e^3}$

$= \frac{1}{2}(\ln e^3)^2 - \frac{1}{2}(\ln e)^2$

$= \frac{1}{2}(3)^2 - \frac{1}{2}(1)^2$

$= \frac{1}{2}(9) - \frac{1}{2} = \frac{9}{2} - \frac{1}{2} = \frac{8}{2} = \boxed{4}$

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OR instead of changing the variable back to x, you can change the limits to "u".

$$\int_1^3 u\,du$$

$$\frac{1}{2}u^2\Big|_1^3$$

$u = \ln x$

limits

$x = e^3 \Rightarrow u = \ln e^3$
$u = 3$

$x = e \Rightarrow u = \ln e$
$u = 1$

$$\frac{1}{2}(3)^2 - \frac{1}{2}(1)^2$$

$$\frac{9}{2} - \frac{1}{2} = \boxed{4}$$

If the $\int$ is interms of "u" the limits must be "u" limits

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Ex2. If you use the u-substitution u=lnx, which of the following integrals is

equivalent to $\int_e^{e^3} \frac{\ln x}{x} dx$

a.) $\int_e^{e^3} u\,du$

b.) $\int_1^3 u\,du$

c.) $\int_e^{e^3} \frac{1}{u} du$

d.) $\int_1^3 \frac{1}{u} du$

e.) $\int_e^{e^3} \ln u\,du$

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Homework

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